EXERCISE -2.1

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of $x$ and $y$.

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$.

$$\frac{x}{3}+1=\frac{5}{3}$$
$$\Rightarrow \frac{x}{3}=\frac{5}{3}-1 \quad y-\frac{2}{3}=\frac{1}{3}$$
$$\Rightarrow \frac{x}{3}=\frac{2}{3} \quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$$
$$\Rightarrow x=2 \quad \Rightarrow y=1$$

$\therefore x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A × B)

= (Number of elements in A) × (Number of elements in B)

= 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

**Question 3:**

If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as

P × Q = {($p$, $q$): $p$∈ P, $q$ ∈ Q}

∴G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

**Question 4:**

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {$m$, $n$} and Q = {$n$, $m$}, then P × Q = {($m$, $n$), ($n$, $m$)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs ($x$, $y$) such that $x$∈ A and $y$ ∈ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

(i) False

If P = {$m$, $n$} and Q = {$n$, $m$}, then

P × Q = {($m$, $m$), ($m$, $n$), ($n$, $m$), ($n$, $n$)}

(ii) True

(iii) True

**Question 5:**

If A = {–1, 1}, find A × A × A.

It is known that for any non-empty set A, A × A × A is defined as

A × A × A = {($a$, $b$, $c$): $a$, $b$, $c \in$ A}

It is given that A = {–1, 1}

∴ A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1),

(1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

Question 6:

If A × B = {($a$, $x$), ($a$, $y$), ($b$, $x$), ($b$, $y$)}. Find A and B.

It is given that A × B = {($a$, $x$), ($a$, $y$), ($b$, $x$), ($b$, $y$)}

We know that the Cartesian product of two non-empty sets P and Q is defined as P × Q = {($p$,$q$): $p \in$ P, $q \in$ Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {$a$, $b$} and B = {$x$, $y$}

Question 7:

Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × C is a subset of B × D

(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)

We have B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ

∴L.H.S. = A × (B ∩ C) = A × Φ = Φ

A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

∴ R.H.S. = (A × B) ∩ (A × C) = Φ

∴L.H.S. = R.H.S

Hence, A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) To verify: A × C is a subset of B × D

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A × C are the elements of set B × D.

Therefore, A × C is a subset of B × D.

Question 8:

Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

A = {1, 2} and B = {3, 4}

∴A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ $n$(A × B) = 4

We know that if C is a set with $n$(C) = $m$, then $n$[P(C)] = $2^m$.

Therefore, the set A × B has $2^4$ = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},

{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

**Question 9:**

Let A and B be two sets such that $n(\text{A}) = 3$ and $n(\text{B}) = 2$. If $(x, 1)$, $(y, 2)$, $(z, 1)$ are in A × B, find A and B, where $x$, $y$ and $z$ are distinct elements.

It is given that $n(\text{A}) = 3$ and $n(\text{B}) = 2$; and $(x, 1)$, $(y, 2)$, $(z, 1)$ are in A × B.

We know that A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B.

∴ $x$, $y$, and $z$ are the elements of A; and 1 and 2 are the elements of B.

Since $n(\text{A}) = 3$ and $n(\text{B}) = 2$, it is clear that A = $\{x, y, z\}$ and B = {1, 2}.

**Question 10:**

The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

We know that if $n(\text{A}) = p$ and $n(\text{B}) = q$, then $n(\text{A} \times \text{B}) = pq$.

$\therefore n(\text{A} \times \text{A}) = n(\text{A}) \times n(\text{A})$

It is given that $n(\text{A} \times \text{A}) = 9$

$\therefore n(\text{A}) \times n(\text{A}) = 9$

$\Rightarrow n(\text{A}) = 3$

The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.

We know that A × A = $\{(a, a): a \in \text{A}\}$. Therefore, –1, 0, and 1 are elements of A.

Since $n(\text{A}) = 3$, it is clear that A = {–1, 0, 1}.

The remaining elements of set A × A are (–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), and (1, 1)

EXERCISE -2.2

Question 1:

Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {($x$, $y$): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.

The relation R from A to A is given as

R = {($x$, $y$): $3x - y = 0$, where $x, y \in$ A}

i.e., R = {($x$, $y$): $3x = y$, where $x, y \in$ A}

∴R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴Domain of R = {1, 2, 3, 4}

The whole set A is the codomainof the relation R.

∴Codomain of R = A = {1, 2, 3, …, 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴Range of R = {3, 6, 9, 12}

Question 2:

Define a relation R on the set **N** of natural numbers by R = {($x$, $y$): $y = x + 5$, $x$ is a natural number less than 4; $x, y \in \mathbf{N}$}. Depict this relationship using roster form. Write down the domain and the range.

R = {($x$, $y$): $y = x + 5$, $x$ is a natural number less than 4, $x, y \in \mathbf{N}$}

The natural numbers less than 4 are 1, 2, and 3.

∴R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴ Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴ Range of R = {6, 7, 8}

**Question 3:**

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {($x$, $y$): the difference between $x$ and $y$ is odd; $x \in$ A, $y \in$ B}. Write R in roster form.

A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {($x$, $y$): the difference between $x$ and $y$ is odd; $x \in$ A, $y \in$ B}

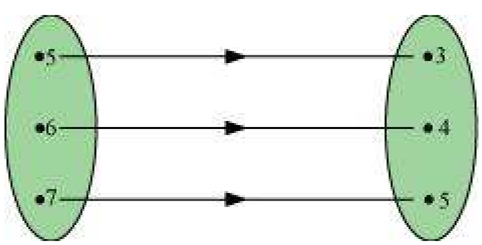
∴R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

**Question 4:**

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?




According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {($x$, $y$): $y = x - 2$; $x \in$ P} or R = {($x$, $y$): $y = x - 2$ for $x$ = 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

**Question 5:**

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{($a$, $b$): $a$, $b \in$ A, $b$ is exactly divisible by $a$}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

A = {1, 2, 3, 4, 6}, R = {($a$, $b$): $a$, $b \in$ A, $b$ is exactly divisible by $a$}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

**Question 6:**

Determine the domain and range of the relation R defined by R = {($x$, $x$ + 5): $x \in$ {0, 1, 2, 3, 4, 5}}.

R = {($x$, $x$ + 5): $x \in$ {0, 1, 2, 3, 4, 5}}

$\therefore$ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

$\therefore$Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

**Question 7:**

Write the relation R = {($x$, $x^3$): $x$ is a prime number less than 10} in roster form.

R = {($x$, $x^3$): $x$ is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore$R = {(2, 8), (3, 27), (5, 125), (7, 343)}

**Question 8:**

Let A = {$x$, $y$, z} and B = {1, 2}. Find the number of relations from A to B.

It is given that A = {$x$, $y$, z} and B = {1, 2}.

∴ A × B = {($x$, 1), ($x$, 2), ($y$, 1), ($y$, 2), ($z$, 1), ($z$, 2)}

Since $n$(A × B) = 6, the number of subsets of A × B is $2^6$.

Therefore, the number of relations from A to B is $2^6$.

**Question 9:**

Let R be the relation on **Z** defined by R = {($a$, $b$): $a$, $b$ ∈ **Z**, $a$ – $b$ is an integer}. Find the domain and range of R.

R = {($a$, $b$): $a$, $b$ ∈ **Z**, $a$ – $b$ is an integer}

It is known that the difference between any two integers is always an integer.

∴Domain of R = **Z**

Range of R = **Z**

## EXERCISE -2.3

**Question 1:**

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

**Question 2:**

Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9 - x^2}$

(i) $f(x) = -|x|, x \in R$

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in \mathbf{R}$, the domain of $f$ is **R.**

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

∴The range of $f$ is $(-\infty, 0]$.

(ii) $f(x)=\sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or $[-3, 3]$.

For any value of $x$ such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3.

∴The range of $f(x)$ is $\{x: 0 \le x \le 3\}$ or $[0, 3]$.

**Question 3:**

A function $f$ is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

The given function is $f(x) = 2x - 5$.

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

**Question 4:**

The function '$t$' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(\mathrm{C})=\frac{9\mathrm{C}}{5}+32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(\mathrm{C}) = 212$

The given function is $t(\mathrm{C})=\frac{9\mathrm{C}}{5}+32$.

Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that $t(\mathrm{C}) = 212$

$$\therefore 212=\frac{9\mathrm{C}}{5}+32$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow 9C=180\times 5$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of $t$, when $t(\mathrm{C}) = 212$, is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$.

(ii) $f(x) = x^2 + 2$, $x$, is a real number.

(iii) $f(x) = x$, $x$ is a real number

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

| $x$ | 0.01 | 0.1 | 0.9 | 1 | 2 | 2.5 | 4 | 5 | … |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.97 | 1.7 | –0.7 | –1 | –4 | –5.5 | –10 | –13 | … |

Thus, it can be clearly observed that the range of $f$ is the set of all real numbers less than 2.

i.e., range of $f = (-\infty, 2)$

**Alter:**

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, $x$, is a real number

The values of $f(x)$ for various values of real numbers $x$ can be written in the tabular form as

| $x$ | 0 | ±0.3 | ±0.8 | ±1 | ±2 | ±3 | | … |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2 | 2.09 | 2.64 | 3 | 6 | 11 | | ….. |

Thus, it can be clearly observed that the range of $f$ is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

**Alter:**

Let $x$ be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

(iii) $f(x) = x$, $x$ is a real number

It is clear that the range of $f$ is the set of all real numbers.

$\therefore$ Range of $f = \mathbf{R}$